PS908
Research Methods & Statistics in Psychology

Analysis of Variance (1)
ANOVA First Steps

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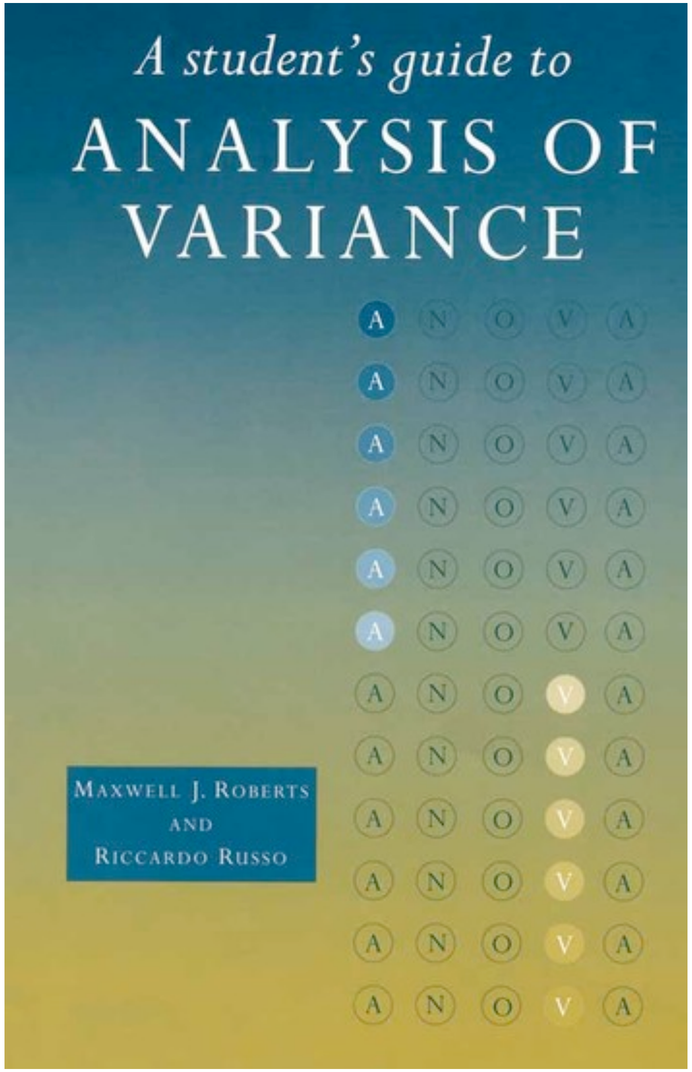
Suggested Text



Topic 1: ANOVA First Steps

- Sources of variability in data
- Measuring data variability
- Types of variance
- Implications of variance for statistics
- Introduction to Analysis of Variance
- Single-factor between-subjects Analysis of Variance
- Interpreting the ANOVA table
- Simplifying the notation

Sources of Variability in Data

- Investigating memory strategies
    - One between-subjects factor with two levels

	Mnemonic Group	No Mnemonic Group
	11	11
	12	8
	15	10
	12	10
	10	11
Level Means	**12**	**10**
Grand Mean	**11**	

Grand Mean is the mean of all scores

Sources of Variability in Data

- Investigating memory strategies
  - Few people obtained the same scores
  - Even people treated ***identically***

	Mnemonic Group	No Mnemonic Group
	11	11
	12	8
	15	10
	12	10
	10	11
Level Means	**12**	**10**
Grand Mean	**11**	

Grand Mean is the mean of all scores

Sources of Variability in Data

- Why might scores be different from each other
  - *Treatment effects*
    - People treated *identically* should behave *identically*
    - People treated *differently* should behave *differently*
  - *Experimental error*
    - In cold hard reality, people treated identically do not necessarily behave identically
    - *Individual differences*
      - People naturally differ in levels of attainment
    - *Random/residual errors*
      - Perfect measurement of true ability is never possible

Measuring Variability

- Informal descriptions: *variability*, *variety*, *distances*
- Mathematical measurement: *variance*, *standard deviation*
- All are conceptually the same

Smaller	**14 16**	**5 6 7**	**2 2 3 4 4**
Larger	**10 20**	**4 5 9**	**1 2 3 4 5**
Largest	**5 25**	**1 8 9**	**1 1 3 5 5**

Measuring Variability

- Variance, standard deviation:
  same mathematical principles for both
- Arithmetic mean is the baseline,
  compare each data point to the baseline

$$\textit{Variance} = \frac{\Sigma(x - \bar{x})^2}{N - 1} \qquad \textit{Standard Deviation} = \sqrt{\frac{\Sigma(x - \bar{x})^2}{N - 1}}$$

X is a set of scores
$\bar{X}$ is the mean of the scores
N is the number of scores

Measuring Variability

$$\textit{Variance} = \frac{\Sigma(x - \bar{x})^2}{N - 1} \qquad \textit{Standard Deviation} = \sqrt{\frac{\Sigma(x - \bar{x})^2}{N - 1}}$$

X is a set of scores
$\bar{X}$ is the mean of the scores
N is the number of scores

Variability:	Variability:
2	1
3	3
4	5

Measuring Variability

- Two important components for the variance equation

$$\sum (x - \bar{x})^2$$

Total variability in the data

$$\text{Variance} = \frac{\textit{Sum of Squares}}{\textit{Degrees of Freedom}}$$

$N - 1$

Gives ***mean***
variability of the data

Measuring Variability

- Two ways to calculate the Sum of Squares
- Second method permits some numerical short cuts later

$$\textit{Sum of Squares} = \Sigma(x - \bar{x})^2$$

$$\textit{Sum of Squares} = \Sigma x^2 - \frac{(\Sigma x)^2}{N}$$

$\Sigma(x^2)$

but usually written
without the brackets

Measuring Variability

$$\text{Sum of Squares} = \Sigma(x - \bar{x})^2$$

$$\text{Sum of Squares} = \Sigma x^2 - \frac{(\Sigma x)^2}{N}$$

Sum of Squares:

2

3

4

Sum of Squares:

2

3

4

Types of Variance

- **Between-Group Variance**
  - The extent to which groups differ *overall*
  - Related to the distance(s) between the level means
  - Caused by *treatment effects* and *experimental error*

	Mnemonic Group	No Mnemonic Group
	11	11
	12	8
	15	10
	12	10
	10	11
Level Means	**12**	**10**
Grand Mean	**11**	

Types of Variance

- **Within-Group Variance**
  - The extent to which scores *within* groups differ
  - Related to the variability of scores inside each group
  - Caused by *experimental error* ***only***

	Mnemonic Group		No Mnemonic Group
	11		11
	12		8
	15		10
	12		10
	10		11
Level Means	**12**		**10**
Grand Mean		**11**	

Implications of Variance for Statistics

- Measure each type of variance, give it numerical values
- Relative sizes of variance are informative

  - Within-group variance = ***X*** *[**X** = any number you like]*
    - Only *experimental error* causes within-group variance
    - Experimental error caused ***X*** variance in your data
  - Between-group variance also = ***X***
    - *Treatment effects* and *experimental error* cause between-group variance
    - Treatment effects added zero to the data, experimental error caused the between group variance and nothing else

Implications of Variance for Statistics

- Measure each type of variance, give it numerical values
- Relative sizes of variance are informative

  - Within-group variance = ***X*** *[**X** = any number you like]*
    - Only *experimental error* causes within-group variance
    - Experimental error caused ***X*** variance in your data
  - Between-group variance = ***more than X***
    - *Treatment effects* and *experimental error* cause between-group variance
    - Data contain more than experimental error: treatment effects have influenced the values too

Implications of Variance for Statistics

- Measure each type of variance, give it numerical values
- Relative sizes of variance are informative

  - Within-group variance = ***X***   *[**X** = any number you like]*
  - Between-group variance = ***not much more than X***
    - Treatment effects are weak, **cannot** infer that they have made a significant contribution to your data
  - Between-group variance = ***much more than X***
    - Treatment effects are strong, **can** infer that they have made a significant contribution to your data

Introduction to Analysis of Variance

- Gives a statistical test-value called *F*
- *F* calculated as the ***ratio*** between two variance values
- *F* always has the same general formula

$$F = \frac{\text{Variance}_{\text{EFFECT}}}{\text{Variance}_{\text{ERROR}}}$$

the ***error term***

Introduction to Analysis of Variance

- *F* ratio for a single factor between-subjects design

$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

- As per …

$$F = \frac{[\text{treatment effects } \textit{plus} \text{ experimental error}]}{[\text{experimental error}]}$$

Introduction to Analysis of Variance

- Theoretical range of the *F* ratio

$$F = \frac{[\text{treatment effects } \textit{plus} \text{ experimental error}]}{[\text{experimental error}]}$$

- Experiment was a complete failure
  - Experimental treatment had no influence on the data

$$F = \frac{[0 + \text{experimental error}]}{[\text{experimental error}]}$$

- ***F* ratio = 1**

Introduction to Analysis of Variance

- Theoretical range of the *F* ratio

$$\boldsymbol{F} = \frac{\text{[treatment effects } \textit{plus} \text{ experimental error]}}{\text{[experimental error]}}$$

- Experiment was a perfect success
    - Everyone treated the same way behaved the same way
    - Everyone treated differently behaved differently

$$\boldsymbol{F} = \frac{\text{[treatment effects + 0]}}{\text{[0]}}$$

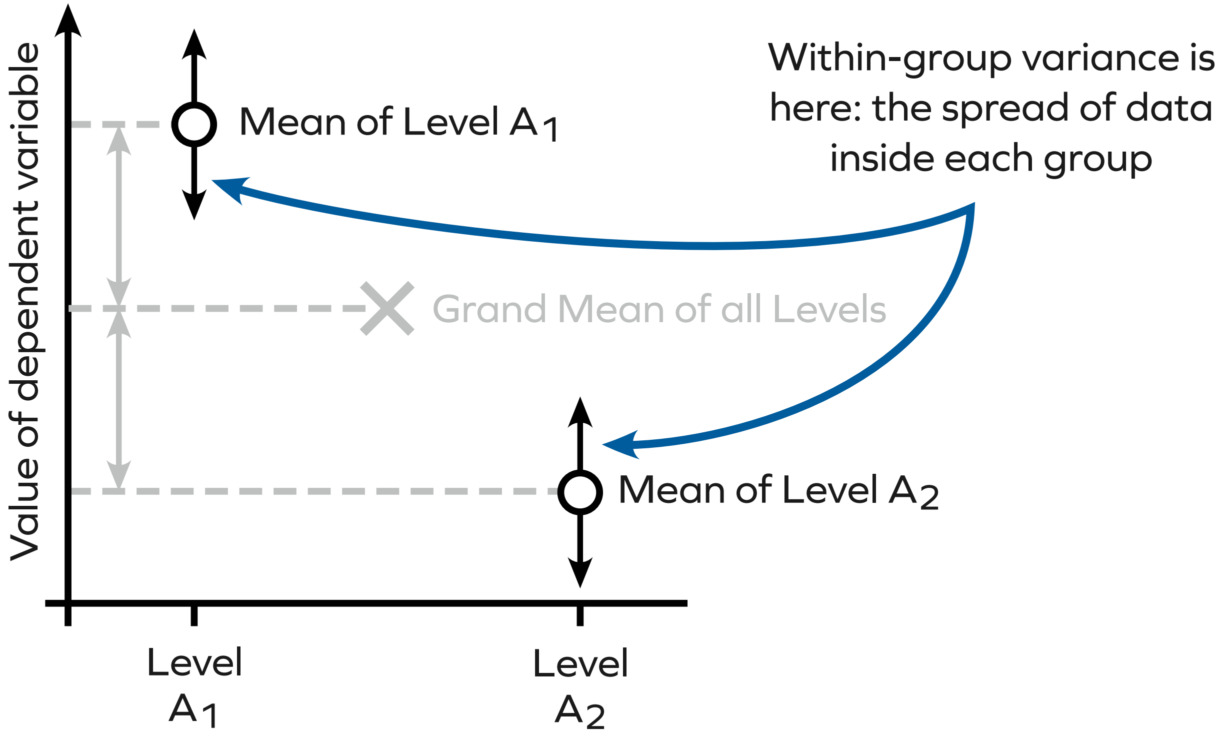
- ***F* ratio = undefined / a very large number / ∞**

Single-Factor Between-Ss ANOVA

- Calculating within-group Sum of Squares (variability)
- Calculating between-group Sum of Squares (variability)
- Calculating degrees of freedom
  (within- and between- group)
- Assembling the components

Single-Factor Between-Ss ANOVA

- Location of the within-group variance



Single-Factor Between-Ss ANOVA

- Within-group variance
  - Find the total variability ***within*** each group (sums of squares)
  - Add together to get the total within-group variability for whole experiment
  - Divide by Degrees of Freedom to get *mean variability*: the ***variance***

$$\frac{[\text{SS Level } A_1] + [\text{SS Level } A_2] + [\text{and so on}]}{[\text{df Level } A_1] + [\text{df Level } A_2] + [\text{and so on}]}$$

Single-Factor Between-Ss ANOVA

- Within-group Sum of Squares

$$SS_{\text{WITHIN}} = \Sigma(A_1 - \overline{A_1})^2 + \Sigma(A_2 - \overline{A_2})^2 + [\text{etc}]$$

Single-Factor Between-Ss ANOVA

- Within-group Sum of Squares

$$SS_{\text{WITHIN}} = \Sigma(A_1 - \overline{A_1})^2 + \Sigma(A_2 - \overline{A_2})^2 + [\text{etc}]$$

but, in general

$$\Sigma(x - \bar{x})^2 = \Sigma x^2 - \frac{(\Sigma x)^2}{N}$$

so that

$$SS_{\text{WITHIN}} = \Sigma A_1^2 - \frac{(\Sigma A_1)^2}{N_{A_1}} + \Sigma A_2^2 - \frac{(\Sigma A_2)^2}{N_{A_2}}$$

Single-Factor Between-Ss ANOVA

- Within-group Sum of Squares

$$SS_{WITHIN} = \Sigma A_1^2 - \frac{(\Sigma A_1)^2}{N_{A_1}} + \Sigma A_2^2 - \frac{(\Sigma A_2)^2}{N_{A_2}}$$

Y = all the datapoints, ignoring which level they are in

$$\Sigma A_1^2 + \Sigma A_2^2 = \Sigma Y^2$$

s = the number of scores in the smallest component of the experiment

With equally-sized levels $N_{A_1} = N_{A_2} = \mathbf{s}$

$$\boxed{SS_{WITHIN} = \Sigma Y^2 - \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2}{s}}$$

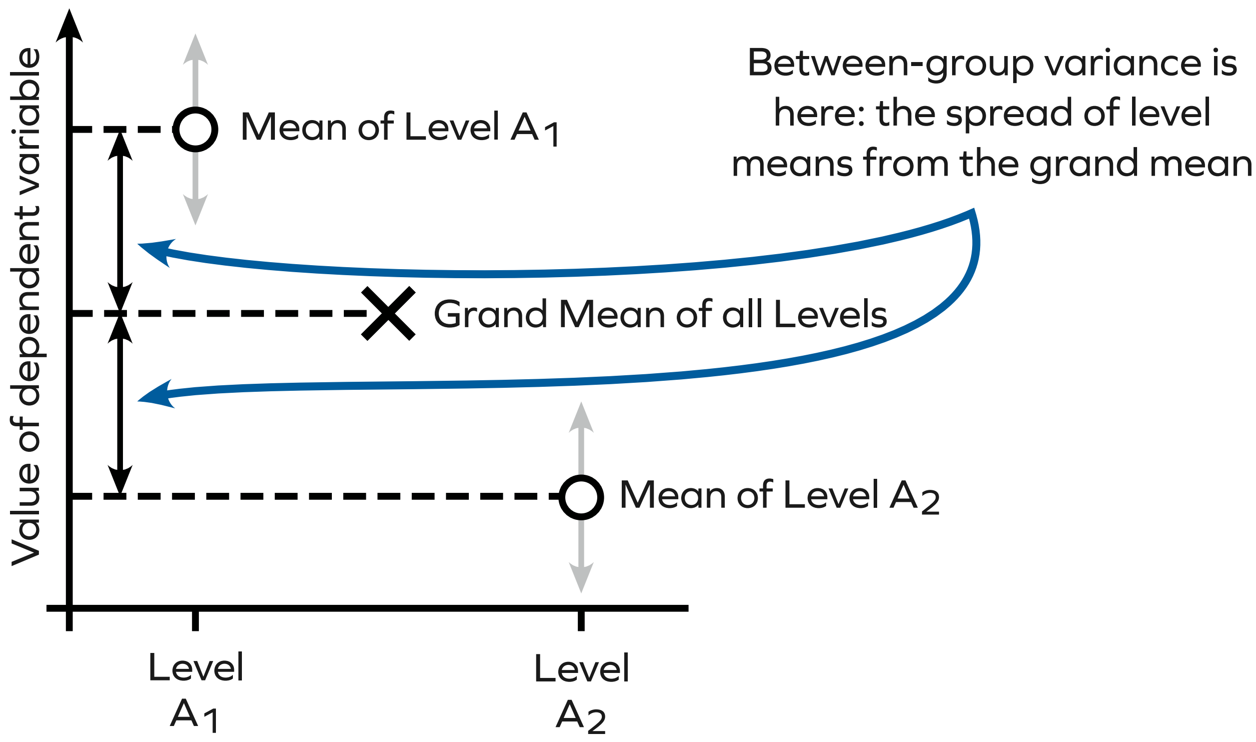
Single-Factor Between-Ss ANOVA

	Mnemonic Group [Level A_1]	No Mnemonic Group [Level A_2]
	11	11
	12	8
	15	10
	12	10
	10	11
Level Means	**12**	**10**
Grand Mean	**11**	

$$SS_{WITHIN} = \Sigma Y^2 - \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2}{s}$$

Single-Factor Between-Ss ANOVA

- Location of the between-group variance



Single-Factor Between-Ss ANOVA

- Between-group variance
  - Find the total variability ***between*** each group mean and the grand mean (sums of squares)
  - Add together to get the total between-group variability for whole experiment
  - Divide by Degrees of Freedom to get *mean variability*: the ***variance***

Single-Factor Between-Ss ANOVA

- Between-group Sum of Squares

$$SS_{BETWEEN} = N_{A_1}(\overline{A_1} - \overline{Y})^2 + N_{A_2}(\overline{A_2} - \overline{Y})^2 + [etc]$$

With equally-sized levels $N_{A_1} = N_{A_2} = s$

Too many steps to show the conversion

$$SS_{BETWEEN} = \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2}{s} - \frac{(\Sigma Y)^2}{N}$$

a = the number of levels in Factor A

$$\boxed{SS_{BETWEEN} = \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2}{s} - \frac{(\Sigma Y)^2}{as}}$$

Single-Factor Between-Ss ANOVA

	Mnemonic Group [Level A_1]	No Mnemonic Group [Level A_2]
	11	11
	12	8
	15	10
	12	10
	10	11
Level Means	**12**	**10**
Grand Mean	**11**	

$$SS_{BETWEEN} = \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2}{s} - \frac{(\Sigma Y)^2}{as}$$

Single-Factor Between-Ss ANOVA

- Degrees of Freedom, df
  - The number of values that are
    free to vary given that their mean is known

Single-Factor Between-Ss ANOVA

- Degrees of Freedom, df
  - The number of values that are *free* to vary given that their mean is known

  - Between-group variance
    - Calculated from **level means** varying with respect to **grand mean**
    - $df_{BETWEEN}$ = number of levels –1
    - $df_{BETWEEN} = (a - 1)$

      where

      a = number of levels in Factor A

Single-Factor Between-Ss ANOVA

- Degrees of Freedom, df
    - The number of values that are ***free*** to vary given that their mean is known
    - Within-group variance
        - Calculated from the **scores** in each level in relation to their **level mean**, aggregated over all levels
        - df_{WITHIN} = (no. of scores in each level –1) x no. of levels
        - $df_{WITHIN} = a(s - 1)$
        - where
          a = number of levels in Factor A
          s = number of scores in each level (number of scores in the **smallest component** of the design)

Single-Factor Between-Ss ANOVA

	Mnemonic Group [Level A_1]	No Mnemonic Group [Level A_2]
	11	11
	12	8
	15	10
	12	10
	10	11
Level Means	**12**	**10**
Grand Mean	**11**	

$df_{BETWEEN} = (a - 1)$

$df_{WITHIN} = a(s - 1)$

a = the number of levels in Factor A

s = the number of scores in each level of Factor A

Single-Factor Between-Ss ANOVA

- Assembling the pieces

$$\text{Variance} = \frac{\text{Sum of Squares}}{\text{Degrees of Freedom}}$$

- Between-group variance:

- Within-group variance:

$$\boldsymbol{F} = \frac{\text{between-group variance}}{\text{within-group variance}}$$

- $\boldsymbol{F} =$

Interpreting the ANOVA Table

- ANOVA table has separate rows for each component

Source	Sum of Squares	Degrees of Freedom	Variance (Mean Square)	*F*-value	*p*-value (sig. level)
A BETWEEN-GROUP					
S/A WITHIN-GROUP					
TOTAL					

- **Source**: the source of variance
  - $\mathbf{SS}_{\mathrm{BETWEEN}}$ abbreviated to $\mathbf{SS}_{\mathrm{A}}$ and $\mathbf{df}_{\mathrm{BETWEEN}}$ to $\mathbf{df}_{\mathrm{A}}$
  - $\mathbf{SS}_{\mathrm{WITHIN}}$ abbreviated to $\mathbf{SS}_{\mathrm{S/A}}$ and $\mathbf{df}_{\mathrm{WITHIN}}$ to $\mathbf{df}_{\mathrm{S/A}}$
- **Variance**: often abbreviated to **Mean Square**

Interpreting the ANOVA Table

- ***Error term*** is the measure of the experimental error

Source	Sum of Squares	Degrees of Freedom	Variance (Mean Square)	*F*-value	*p*-value (sig. level)
A BETWEEN-GROUP	10	1	10	4	
S/A WITHIN-GROUP	20	8	2.5		
TOTAL	30	9			

$$F = \frac{\text{Variance}_{\text{EFFECT}}}{\text{Variance}_{\text{ERROR}}}$$

- Within-group variance for a between-subjects ANOVA

Interpreting the ANOVA Table

- Is the *F* value large enough to be significant?

Source	Sum of Squares	Degrees of Freedom	Variance (Mean Square)	*F*-value	*p*-value (sig. level)
A BETWEEN-GROUP	10	1	10	4	
S/A WITHIN-GROUP	20	8	2.5		
TOTAL	30	9			

- *F* values have TWO degrees of freedom
  - **(between-group variance, within-group variance)**
- Observed $F(1,8) = 4$
- Check with computer or tables of critical values

Interpreting the ANOVA Table

- ACROSS 1 then DOWN 8

Critical Values of the F *Distribution*

.05 significance level in bold type

.01 significance level in plain type

degrees of freedom of numerator

edom of denominator

	1	2	3	4	5
1	**161**	**200**	**216**	**225**	**230**
	4052	4999	5403	5625	5764
2	**18.5**	**19.0**	**19.2**	**19.2**	**19.3**
	98.5	99.0	99.2	99.2	99.3
3	**10.1**	**9.55**	**9.28**	**9.12**	**9.01**
	34.1	30.8	29.5	28.7	28.2
4	**7.71**	**6.94**	**6.59**	**6.39**	**6.26**
	21.2	18.0	16.7	16.0	15.5
5	**6.61**	**5.79**	**5.41**	**5.19**	**5.05**
	16.3	13.3	12.1	11.4	11.0
6	**5.99**	**5.14**	**4.76**	**4.53**	**4.39**
	13.8	10.9	9.78	9.15	8.75
7	**5.59**	**4.74**	**4.35**	**4.12**	**3.97**
	12.2	9.55	8.45	7.85	7.46
8	**5.32**	**4.46**	**4.07**	**3.84**	**3.69**
	11.3	8.65	7.59	7.01	6.63
9	**5.12**	**4.26**	**3.86**	**3.63**	**3.48**
	10.6	8.02	6.99	6.42	6.06
10	**4.96**	**4.10**	**3.71**	**3.48**	**3.33**
	10.0	7.56	6.55	5.99	5.64
11	**4.84**	**3.98**	**3.59**	**3.36**	**3.20**
	9.65	7.21	6.22	5.67	5.32

Interpreting the ANOVA Table

- Critical *F*, df = (1,8), at $p < .05$ significance level = 5.32

Source	Sum of Squares	Degrees of Freedom	Variance (Mean Square)	*F*-value	*p*-value (sig. level)
A BETWEEN-GROUP	10	1	10	4	$p > .05$ NS
S/A WITHIN-GROUP	20	8	2.5		
TOTAL	30	9			

- Observed $F(1,8) = 4$
- Non-significant effect of memory strategy, $p > .05$

Interpreting the ANOVA Table

- No significant difference between means for Mnemonic versus No Mnemonic groups

	Mnemonic Group		No Mnemonic Group
	11		11
	12		8
	15		10
	12		10
	10		11
Level Means	**12**		**10**
Grand Mean		**11**	

Grand Mean is the mean of all scores

Simplifying the Notation

- Summary so far: Sums of Squares

$$SS_{\text{BETWEEN}} = \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2}{s} - \frac{(\Sigma Y)^2}{as}$$

$$SS_{\text{WITHIN}} = \Sigma Y^2 - \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2}{s}$$

$$SS_{\text{TOTAL}} = \Sigma Y^2 - \frac{(\Sigma Y)^2}{as}$$

$$SS_{\text{TOTAL}} = SS_{\text{BETWEEN}} + SS_{\text{WITHIN}}$$

Simplifying the Notation

- Summary so far: Degrees of Freedom

$$df_{BETWEEN} = (a - 1)$$

$$df_{WITHIN} = a(s - 1)$$

$$df_{TOTAL} = (as) - 1$$

$$df_{TOTAL} = df_{BETWEEN} + df_{WITHIN}$$

Simplifying the Notation

$$SS_{BETWEEN} = \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2}{s} - \frac{(\Sigma Y)^2}{as}$$

$$SS_{TOTAL} = \Sigma Y^2 - \frac{(\Sigma Y)^2}{as}$$

$$\frac{(\Sigma Y)^2}{as}$$

$$\frac{(\text{the Grand Total})^2}{\text{the number of scores that make up the grand total}}$$

Simplifying the Notation

$$SS_{BETWEEN} = \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2}{s} - \frac{(\Sigma Y)^2}{as}$$

$$SS_{WITHIN} = \Sigma Y^2 - \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2}{s}$$

$$\frac{(\Sigma A_1)^2 + (\Sigma A_2)^2}{s}$$

$$\frac{(\text{Level Total } A_1)^2 + (\text{Level Total } A_2)^2}{\text{the number of scores that make up EACH level total}}$$

Simplifying the Notation

$$SS_{WITHIN} = \Sigma Y^2 - \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2}{s}$$

$$SS_{TOTAL} = \Sigma Y^2 - \frac{(\Sigma Y)^2}{as}$$

$$\Sigma Y^2$$

$$\frac{(Score_1)^2 + (Score_2)^2 + (Score_3)^2 + (Score_4)^2 + (Score_5)^2 \ \text{etc.}}{1\text{: only one score makes up each square}}$$

Simplifying the Notation

- All equations have the same basic form

$$[X] = \text{the } \textit{Basic Ratio} \text{ of } \boldsymbol{X}$$

$$\frac{\text{The sum of (each and every } X)^2}{\text{the number of scores that made up each individual } \boldsymbol{X}}$$

Simplifying the Notation

- Three Basic Ratios for a between-subjects design

$[T]$ = The Basic Ratio of the grand total

$$\frac{(\Sigma Y)^2}{as}$$

$[A]$ = The Basic Ratio of the level totals

$$\frac{(\Sigma A_1)^2 + (\Sigma A_2)^2}{s}$$

$[Y]$ = The Basic Ratio of the individual scores

$$\Sigma Y^2$$

Simplifying the Notation

- Basic Ratio of the *Grand Total*

[*T*] = The Basic Ratio of the grand total as $\dfrac{(\Sigma Y)^2}{as}$

$$\frac{(\text{Grand Total})^2}{\text{the number of scores that make up the grand total}}$$

	Mnemonic Group [Level A_1]	No Mnemonic Group [Level A_2]
Individual Scores	11	11
	12	8
	15	10
	12	10
	10	11
Level TOTALs	**60**	**50**
Grand TOTAL	**110**	

$$[T] = \frac{110^2}{10} = \frac{12100}{10} = 1220$$

Simplifying the Notation

- Basic Ratio of the *Level Totals*

$[A]$ = The Basic Ratio of the level totals $\dfrac{(\Sigma A_1)^2 + (\Sigma A_2)^2}{s}$

$$\frac{(\text{Level Total } A_1)^2 + (\text{Level Total } A_2)^2 \text{ etc.}}{\text{the number of scores that make up EACH \textbf{level} total}}$$

	Mnemonic Group [Level A_1]	No Mnemonic Group [Level A_2]
Individual Scores	11	11
	12	8
	15	10
	12	10
	10	11
Level TOTALs	**60**	**50**
Grand TOTAL	**110**	

$$[A] = \frac{60^2 + 50^2}{5} = \frac{3600+2500}{5} = \frac{6100}{5} = 1220$$

Simplifying the Notation

- Basic Ratio of the *Individual Scores*

[*Y*] = The Basic Ratio of the individual scores

$$\Sigma Y^2$$

$$\frac{(Y_1)^2+(Y_2)^2+(Y_3)^2+(Y_4)^2+(Y_5)^2+(Y_6)^2+(Y_7)^2+(Y_8)^2+(Y_9)^2 \text{ etc.}}{1}$$

	Mnemonic Group [Level A_1]	No Mnemonic Group [Level A_2]
Individual Scores	11	11
	12	8
	15	10
	12	10
	10	11
Level TOTALs	**60**	**50**
Grand TOTAL	**110**	

$$[Y] = 11^2+12^2+15^2+12^2+10^2 +11^2+8^2+10^2+10^2+11^2$$

$$= \begin{matrix} 121+144+225+144+100 \\ +121+64+100+100+121 \end{matrix} = 1240$$

Simplifying the Notation

$$SS_{\text{BETWEEN}} = \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2}{s} - \frac{(\Sigma Y)^2}{as}$$

$$SS_{\text{BETWEEN}} = [A] - [T]$$

$$SS_{\text{WITHIN}} = \Sigma Y^2 - \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2}{s}$$

$$SS_{\text{WITHIN}} = [Y] - [A]$$

$$SS_{\text{TOTAL}} = \Sigma Y^2 - \frac{(\Sigma Y)^2}{as}$$

$$SS_{\text{TOTAL}} = [Y] - [T]$$

Simplifying the Notation

- Between-Group Variance:
  - Variability of the ***level means*** with respect to the ***grand mean***

$$SS_{\text{BETWEEN}} = [A] - [T]$$

$$SS_{\text{BETWEEN}} = \begin{array}{c}\text{Basic Ratio of} \\ \text{the } \textbf{\textit{level totals}}\end{array} - \begin{array}{c}\text{Basic Ratio of} \\ \text{the } \textbf{\textit{grand total}}\end{array}$$

Simplifying the Notation

- Within-Group Variance:
  - Variability of the ***individual scores*** with respect to their ***level means***

$$SS_{WITHIN} = [Y] - [A]$$

$$SS_{WITHIN} = \text{Basic Ratio of the } \textbf{\textit{individual scores}} - \text{Basic Ratio of the } \textbf{\textit{level totals}}$$

Simplifying the Notation

- Total Variance:
  - Variability of the ***individual scores*** with respect to the ***grand total***

$$SS_{\text{TOTAL}} = [Y] - [T]$$

$$SS_{\text{TOTAL}} = \text{Basic Ratio of the } \textbf{\textit{individual scores}} - \text{Basic Ratio of the } \textbf{\textit{grand total}}$$

Simplifying the Notation

	[Level A_1]		[Level A_2]
	1		3
	2		4
	2		4
	3		5
Level Means	**2**		**4**
Grand Mean		**3**	

$[T] =$

$[A] =$

$[Y] =$

Simplifying the Notation

	[Level A_1]		[Level A_2]
	1		3
	2		4
	2		4
	3		5
Level Means	**2**		**4**
Grand Mean		**3**	

$SS_{BETWEEN} = [A] - [T] =$

$SS_{WITHIN} = [Y] - [A] =$

Simplifying the Notation

	[Level A_1]		[Level A_2]
	1		3
	2		4
	2		4
	3		5
Level Means	**2**		**4**
Grand Mean		**3**	

$df_{BETWEEN} = (a - 1) =$

$df_{WITHIN} = a(s - 1) =$

Simplifying the Notation

- The blank ANOVA table, as before

Source	Sum of Squares	Degrees of Freedom	Variance (Mean Square)	*F*-value	*p*-value (sig. level)
A BETWEEN-GROUP					
S/A WITHIN-GROUP					
TOTAL					

Simplifying the Notation

- Is the *F* value large enough to be significant?

Source	Sum of Squares	Degrees of Freedom	Variance (Mean Square)	*F*-value	*p*-value (sig. level)
A BETWEEN-GROUP	8	1	8	12	
S/A WITHIN-GROUP	4	6	$^{2}/_{3}$		
TOTAL	12	7			

- *F* value has Degrees of Freedom = (1,6)
- Check with computer or tables of critical values

Simplifying the Notation

- ACROSS 1 then DOWN 6

Critical Values of the F *Distribution*

.05 significance level in bold type
.01 significance level in plain type

degrees of freedom of numerator

	1	2	3	4	5
1	**161**	**200**	**216**	**225**	**230**
	4052	4999	5403	5625	5764
2	**18.5**	**19.0**	**19.2**	**19.2**	**19.3**
	98.5	99.0	99.2	99.2	99.3
3	**10.1**	**9.55**	**9.28**	**9.12**	**9.01**
	34.1	30.8	29.5	28.7	28.2
4	**7.71**	**6.94**	**6.59**	**6.39**	**6.26**
	21.2	18.0	16.7	16.0	15.5
5	**6.61**	**5.79**	**5.41**	**5.19**	**5.05**
	16.3	13.3	12.1	11.4	11.0
6	**5.99**	**5.14**	**4.76**	**4.53**	**4.39**
	13.8	10.9	9.78	9.15	8.75
7	**5.59**	**4.74**	**4.35**	**4.12**	**3.97**
	12.2	9.55	8.45	7.85	7.46
8	**5.32**	**4.46**	**4.07**	**3.84**	**3.69**
	11.3	8.65	7.59	7.01	6.63
9	**5.12**	**4.26**	**3.86**	**3.63**	**3.48**
	10.6	8.02	6.99	6.42	6.06
10	**4.96**	**4.10**	**3.71**	**3.48**	**3.33**
	10.0	7.56	6.55	5.99	5.64
11	**4.84**	**3.98**	**3.59**	**3.36**	**3.20**
	9.65	7.21	6.22	5.67	5.32

...edom of denominator

Simplifying the Notation

- Critical *F*, df = (1,6), at $p < .05$ significance level = 5.99

Source	Sum of Squares	Degrees of Freedom	Variance (Mean Square)	*F*-value	*p*-value (sig. level)
A BETWEEN-GROUP	8	1	8	12	$p < .05$ *
S/A WITHIN-GROUP	4	6	$^{2}/_{3}$		
TOTAL	12	7			

- Observed $F(1,6) = 12$
- Significant effect, $p < .05$

Single-Factor Between-Subjects ANOVA

Source	Sum of Squares	Degrees of Freedom	Variance (Mean Square)	*F*-value
A BETWEEN-GROUP	$[A] - [T]$	$(a - 1)$	$\frac{[A] - [T]}{(a - 1)}$	$\frac{\text{VARIANCE}_{A}}{\text{VARIANCE}_{S/A}}$
S/A WITHIN-GROUP	$[Y] - [A]$	$a(s - 1)$	$\frac{[Y] - [A]}{a(s - 1)}$	
TOTAL	$[Y] - [T]$	$(as) - 1$		

- [*T*] is the basic ratio of the grand total
  [*A*] is the basic ratio of the level totals
  [*Y*] is the basic ratio of the individual scores
- a is the number of levels in Factor A
  s is the number of scores in *each level* of Factor A